GEODEICS ON THE COSET SPACES AS A DYNAMICAL REALIZATION OF *l*-CONFORMAL GALILEI ALGEBRA

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ГЕОДЕЗИЧЕСКИЕ НА ФАКТОРПРОСТРАНСТВАХ КАК ДИНАМИЧЕСКИЕ РЕАЛИЗАЦИИ *l*-КОНФОРМНОЙ АЛГЕБРЫ ГАЛИЛЕЯ

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Abstract. *Construction of dynamical realizations of l-conformal Galilei algebra without higher derivative terms is discussed.*

Introduction. In recent years nonrelativistic conformal Galilei algebras attracted considerable interest [2-7]. The conformal extension of the Galilei algebra is parameterized by a (half)integer parameter l [1]. A peculiar feature of this algebra is that it involves acceleration generators along with the standard set of generators of Galilei algebra. Most of the examples of dynamical realizations of this algebra encounters with a problem of the precence of higher derivative terms or functional dependence of the acceleration generators. The main goal of this note is to construct metric on the coset space of l-conformal Galilei group and analyze corresponding geodesics equations. Considering geodesics equations as a dynamical realization, we show that it is free of the problems mentioned above.

l **-conformal Galilei algebra.** The l-conformal Galilei algebra involves the generators of time translation H, dilatation D, special conformal transformation K, spatial rotations M_{ij} (with $i=1,\ldots,d$), spatial translations $C_i^{(0)}$, Galilei boosts $C_i^{(1)}$ and accelerations $C_i^{(\alpha)}$ with $\alpha = 2,\ldots,2l$. The structure relations of the algebra read

$$[H,D]=iH, \quad [H,K]=2iD, \quad [D,K]=iK, \quad [H,C_i^{(n)}]=inC_i^{(n-1)}, \quad [D,C_i^{(n)}]=i(n-l)C_i^{(n)},$$
$$[K,C_i^{(n)}]=i(n-2l)C_i^{(n+1)}, \quad [M_{ij},C_k^{(n)}]=-i\delta_{ik}C_j^{(n)}+i\delta_{jk}C_i^{(n)},$$
$$[M_{ij},M_{kl}]=-i\delta_{ik}M_{jl}-i\delta_{jl}M_{ik}+i\delta_{il}M_{jk}+i\delta_{jk}M_{il}. \qquad (1)$$

We construct a metric on the coset space of l-conformal Galilei group G/H. Let us choose H generated by dilations D and rotations M_{ij} operators. For an element $\tilde{G}$ of the coset space G/H, we define corresponding Maurer-Cartan one-forms by the standard way

$$\tilde{G}^{-1}d\tilde{G}=i(\omega_H H+\omega_K K+\omega_D D+\omega_i^{(n)}C_i^{(n)}). \qquad (2)$$

In what follows we will also need the form of the algebra (1) written in terms of Maurer-Cartan one forms

$$d\omega_H - \omega_H \wedge \omega_D = 0, \quad d\omega_K - \omega_D \wedge \omega_K = 0, \quad d\omega_i^n - n\omega_H \wedge \omega_i^{n-1} - (n-l)\omega_D \wedge \omega_i^n - (n-2l)\omega_K \wedge \omega_i^{n+1} = 0. \quad (3)$$

Geodesics on the coset space as dynamical realization. Maurer-Cartan one-forms on the coset space transform homogeneously under the action of l-conformal Galilei group. Using these one-forms we are able to construct a metric in G/H

$$ds^2 = \omega_H \omega_K + S_{n,m}\omega_i^{(n)}\omega_i^{(m)}. \quad (4)$$

The metric written above is invariant under the action of l-conformal Galilei group, provided by the following restriction on the matrix S_{mn} coefficients

$$S_{m,n}(m+n-2l) = 0, \quad \forall\, m, n. \quad (5)$$

Having fixed the form of the metric, let us write down the action, defining geodesics equations

$$S = \int d\lambda \left(\omega_H \omega_K + S_{n,m}\omega_i^{(n)}\omega_i^{(m)}\right), \quad (6)$$

where it is supposed that all the differentials in Cartan one-forms are replaced by velocities. Varying this action and using structure relations (3), one may obtain geodesics equations written in terms of Cartan one-forms

$$\frac{1}{2}\dot{\omega}_H = -\frac{1}{2}\omega_H\omega_D + (q-2l)S_{p,q+1}\omega^{(p)}\omega^{(q)}, \quad \frac{1}{2}\dot{\omega}_K = \frac{1}{2}\omega_K\omega_D + qS_{p,q-1}\omega^{(p)}\omega^{(q)},$$

$$\dot{\omega}^{(p)}S_{p,n} = -(n-l)S_{p,n}\omega^{(p)}\omega_D - nS_{p,n-1}\omega^{(p)}\omega_H - (n-2l)S_{p,n+1}\omega^{(p)}\omega_K, \quad (7)$$

where dot over the symbol denotes derivative with respect to parapeter λ.

To proceed, we fix the form of the coset representative

$$\widetilde{G} = e^{itH} e^{irK} e^{ix_i^{(n)}C_i^{(n)}}, \quad (8)$$

with coordinates t, r and $x_i^{(n)}$ on the coset space. Corresponding Maurer-Cartan 1—forms (2) read

$$\omega_H = dt, \qquad \omega_K = r^2 dt + dr, \qquad \omega_D = -2rdt, \qquad \omega_i^{(n)} = dx_i^{(n)} + a_i^{(n)}dt + b_i^{(n)}dr, \quad (9)$$

$$a_i^{(n)} = 2r(n-l)x_i^{(n)} - (n+1)x_i^{(n+1)} - r^2(n-2l-1)x_i^{(n-1)}, \quad b_i^{(n)} = -(n-2l-1)x_i^{(n-1)}.$$

Left multiplication by the group element determines transformations generated by the l-conformal group

$$H = \frac{\partial}{\partial t}, \; K = t^2\frac{\partial}{\partial t} + (1-2tr)\frac{\partial}{\partial r} - 2t(n-l)x_i^{(n)}\frac{\partial}{\partial x_i^{(n)}}, \quad D = t\frac{\partial}{\partial t} - r\frac{\partial}{\partial r} - (n-l)x_i^{(n)}\frac{\partial}{\partial x_i^{(n)}},$$

$$M_{ij} = x_i^{(n)}\frac{\partial}{\partial x_j^{(n)}} - x_j^{(n)}\frac{\partial}{\partial x_i^{(n)}}, \; C_i^{(m)} = B^{nm}\frac{\partial}{\partial x_i^{(n)}}, \quad B^{mn} = \sum_{s=0}^{m}\frac{(-1)^{n-s}m!(2l-s)!}{s!(m-s)!(n-s)!(2l-n)!}t^{m-s}r^{n-s}, \quad (10)$$

where it is assumed that in the last formula the terms with $s > m$ and $s > n$ vanish. Geodesics equations (7) are invariant under the action of l-conformal group, generated by the vectors written above. In order to construct the corresponding integrals of motion in the explicit form, we redefine the coordinates $x_i^{(n)}$

$$x_i'^{(n)} = (B^{-1})^{np}x_i^{(p)}, \quad (11)$$

with the use of the matrix $(B^{-1})^{np}$ inverse to B^{np}

$$(B^{-1})^{np} = \sum_{q=n}^{2l} \frac{(-1)^{q-n} q!(2l-p)!}{n!(q-p)!(q-n)!(2l-q)!} t^{q-n} r^{q-p}, \tag{12}$$

where it is assumed that the terms with $p > q$ and $n > q$ vanish. The generators of the symmetry transformations in the new coordinate system read

$$H = \frac{\partial}{\partial t} - (n+1)x_i'^{(n+1)} \frac{\partial}{\partial x_i'^{(n)}}, \qquad D = t\frac{\partial}{\partial t} - r\frac{\partial}{\partial r} - (n-l)x_i'^{(n)} \frac{\partial}{\partial x_i'^{(n)}},$$

$$K = t^2 \frac{\partial}{\partial t} + (1-2tr)\frac{\partial}{\partial r} - (n-2l-1)x_i'^{(n-1)} \frac{\partial}{\partial x_i'^{(n)}}, \qquad C_i^{(n)} = \frac{\partial}{\partial x_i'^{(n)}}. \tag{13}$$

Now, we can write the integrals of motion in the explicit form

$$H = 2r^2\dot{t} + \dot{r} - (n+1)x_i'^{(n+1)} C_i^{(n)}, \qquad D = t(2r^2\dot{t} + \dot{r}) - r\dot{t} - (n-l)x_i'^{(n)} C_i^{(n)},$$

$$K = t^2(2r^2\dot{t} + \dot{r}) + (1-2tr)\dot{t} - (n-2l-1)x_i'^{(n-1)} C_i^{(n)}, \quad C_i^{(n)} = B^{pn} S_{p,m} \omega_i^{(m)}. \tag{14}$$

It is straightforward to verify that (14) are the functionally independent integrals of motion for (7).

Conclusion. Let us summarize our results. We constructed a metric on the coset space of *l*-conformal Galilei group. Considering geodesics equations for this space as a dynamical realization, we have shown that the corresponding acceleration generators are independent. Geodesics equations represent second order dynamical system, i.e they do not involve higher derivative terms typical for the most of the examples of dynamical realizations of *l*-conformal algebra. In this regard, it is interesting to investigate for a possible link between dynamical realization discussed in this note and other known realizations possessing l-conformal symmetry.

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